

## A FREQUENCY BASED FREE VIBRATION ANALYSIS OF A HAT STIFFENED PLATE, FOR IDENTIFICATION OF THE DAMAGE

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### ABSTRACT

*Hat section has excellent properties, in bending about its both principal axes. Due to this property, hat section is generally used to increase the strength of the parent structure. Therefore, hat stiffened structures, finds many industrial applications like in automobile, airplane, and naval ships. In all such applications, an inherent structure that finds is a hat stiffened plate. This paper has presented an analysis of diagonally hat stiffened plate structure for the detection of damage caused due to the breaking of spot weld connections. A transformed section method was used to model the structure at intact condition and solved by Euler- Bernoulli's equation to find its natural frequencies. Similarly a model for damaged hat plate structure was prepared. Using the conjugate beam method, a static deflection of lumped masses was determined, to solve the matrix equation of motion. Eigen values extracted from the solution and corresponding natural frequencies were found in the damaged condition. A finite element (FE) model of the hat plate structure was solved by ANSYS package, for both intact and damaged conditions. An experimental modal analysis (EMA) had conducted to validate the results. A good agreement found among the results from all the three approaches.*

**KEYWORDS:** *Hat-Stiffened Plate, Natural Frequencies, Conjugates Beam Method, Modal Analysis & Damage Detection.*

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### 1. INTRODUCTION

Hat sections are generally made of rolled steel. It finds very wide and increasing applications in today's market. When incorporated to parent structure, it enhances the physical properties of the structure. A hat stiffened plate is one of the examples where due to hat section, bending strength of the plate is notably increased. A hat stiffened plate finds applications in various components of automobile like door, floor, bonnet etc. Thus, for designing the optimum structure a prior choice is given to the hat stiffened plates due to its low weight and high bending strength. But one issue that surfaces in such applications is the separation of the hat flanges from the plate. This issue may be solved by properly monitoring the structure. Condition monitoring of the structure can be reliably done with frequency based vibration analysis. Specifically experimental modal analysis that finds the modal parameters like frequency, mode shapes and damping, can be used for damage detection.

As the natural frequency is easily possible to measure, it is predominantly used as a diagnostic parameter for structural health monitoring technique. Cawley and Adams [1] gave a detailed formulation, to detect damage in a structure using differences between damaged and undamaged resonant frequencies for each mode, in comparison with finite element results. Many damage models have been developed by the researchers, the most popular among these is used by Laxmikant Kannappan et.al [2], to identify crack in the cantilever beam. The change in its natural frequency of the beam is determined before and after damage. Here cracks are modeled as a rotational spring with

stiffness  $K$ . When a crack occurs in a beam structure, it causes a reduction in the cross section area of the beam in the concerned plane. Based on this aspect Nakhaei A. et. al. [3] proposed a model for cracked beam, considering all parameters and represented it as a stepped cross section beam. Along with natural frequencies (eigen values) eigenvectors can be used to identify internal defects and cracks in the structure, as shown by Dems K. et. al. [4].

Hat sections are more popular as stiffener to the parent structure. So, hat stiffened plates are studied for the purpose of numerous investigations like Yetman J. E. et.al. [5] asserted that, the structural integrity of the hat stiffened plate is mostly governed by disbond between stiffener and the plate, and is the measure of disbond size and location.

Among various solution techniques the finite element (FE) method has received more preference. Tharian M. et. al. [6] quantified the structural advantages of hat shaped stiffeners, over the commonly used open section stiffeners. For most of the structure dynamic environment is common. So, the study of their dynamic characteristics is vital to develop accurate strategies to control the modal vibrations, as explained by Husain N. A. et.al. [7]. Conventional as well as super finite elements are also used by Shahed J. And Khedmati M [8], to analyze the vibrations of stiffened plate. Prusty G. B. [9,10] carried out an investigation on free vibration and buckling analyses of laminated composite stiffened shell structures, with laminated open section (rectangular or 'T' shaped) and closed section ('hat' shaped) stiffeners. With the use of ANSYS finite element codes, Siddiqui, H et. al. [11] studied the effect of various parameters, such as boundary conditions, aspect ratio of non-dimensional frequency parameter of the plate.

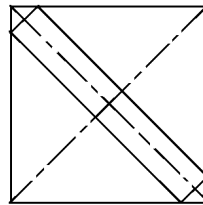
The transformed section method has been certainly a more favorable in structural analysis. It is an alternative for the analysis of complex structures with linearly elastic material. The transformed section method has also been used successfully in different reinforcing materials, cantilever piezoelectric unimorph and composite members by Elsanadedy H. M. et.al. [12], A. M. Matos [13] and Hamilton R. et.al. [14]. A great contribution is made by Mohr G. O. to structural analysis, through a method known as elastic loads or the conjugate beam method is discussed by Rojas A. [16] and Monforton et.al. [17]. An experimental modal analysis [18] has found most reliable and non destructive technique, for damage detection of hat stiffened plate.

The above review of various literatures on vibration analysis of stiffened plate reveals that the concerned work is carried out most preferably with the finite element method. To deal with huge calculations is difficult unless the use of any suitable commercial software package. This may lead to more cost incurred for large and complex structures. So it becomes essential also to have a conventional method based vibration analysis. In this paper a hat section stiffened plate is analyzed for the separation of the stiffener flange from the plate due to cracking of the spot welds which is termed as damage hereafter. The cross section of the intact structure is represented using the transformed section method and then Euler-Bernoulli's equation is used to determine its natural frequencies. When damage is introduced, the section of the model is again defined by transformed section method. This transformed section model is analyzed using conjugate beam method to find the static deflection of the specified lumped masses. A matrix equation is solved to obtain the natural frequencies for damaged condition. The change in frequency magnitude due to the damage has assured the viability of approach. Finite element (FE) method and experimental modal analysis is also used for validation purpose of the proposed method.

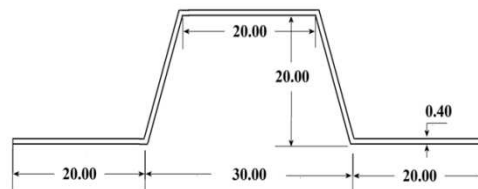
## 2. MATHEMATICAL MODEL

A diagonally hat stiffened plate (300 x 300 mm) as shown in Figure1 is taken for free vibration analysis. The plate

and stiffener are having same thickness and material. The flanges of the stiffener are joined to the plate by spot weld joints applied at centre and 75 mm apart from each other along the flange Length. Thus total 10 spot welds are applied. Transformed section method consists of transforming the cross section of a complex structure into an equivalent cross section of an imaginary beam made of a same material. This imaginary beam with transformed cross section is analyzed by one of the suitable beam theories. Thus the results obtained are said to be applicable for the original structure. The hat section channel has sectional dimensions as shown in Figure 2 and hat stiffened plate properties are given in Table 1.



**Figure 1: Hat Stiffened Plate Model**



**Figure 2: Hat Cross Section of the Stiffener**

**Table 1: Hat Stiffened Plate Properties**

Sr. No.	Parameter	Value
1	Distance of CG from flat face of plate	3.4 mm
2	Moment of inertia	5475 mm <sup>4</sup>
3.	Young's Modulus (E)	200GPa
4.	Material Density ( $\rho$ )	7850 Kg/ m <sup>3</sup>
5.	Poison's Ratio ( $\nu$ )	0.3

To have the transformed section of structure, its neutral axis position and moment resisting capacity must be same as original section. The position of the neutral axis is found by equating net force acting on the cross section to zero. Letting suffix 1 for denoting original and 2 for denoting transformed section,

$$\int (\sigma_b dA)_1 + \int (\sigma_b dA)_2 = 0 \quad (1)$$

Where,  $\sigma_b$  = Bending stress in cross section and  $dA$  = Area of cross section

Using flexural equation for beam,

$$E_1 \int (y dA)_1 + E_2 \int (y dA)_2 = 0 \quad (2)$$

Here,  $E_1, E_2$  = Young's modulus of original section and transformed section respectively

$y$  = Distance of fiber from the neutral axis.

Secondly, from the moment resisting capacity ( $M_r$ ),

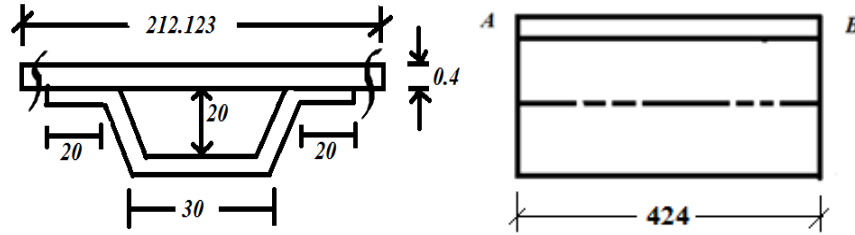
$$(M_{r1}) = (M_{r2}) \gg \left( \frac{\sigma_b I}{y_{max}} \right)_1 = \left( \frac{\sigma_b I}{y_{max}} \right)_2 \quad (3)$$

For the same material as well as same distance of farthest fiber from neutral axis,

$$I_1 = I_2 \quad (4)$$

$I_1, I_2$  = Moment of Inertia (M. I.) of Original and transformed section respectively.

Here, in transformed section, plate is considered of average cross section area having same thickness and moment of inertia as of an original plate (Figure3).



**Figure 3: Orthographic Views of Transformed Beam Model for Intact Condition**

According to Euler- Bernoulli's equation, the free vibration of transformed section beam is,

$$C^2 \frac{\partial^4 w}{\partial x^4}(x, t) + \frac{\partial^4 w}{\partial t^4}(x, t) = 0 \quad (5)$$

$$\text{Where, } C = \sqrt{\frac{EI}{\rho A}} \quad (6)$$

Putting the initial conditions as,

$$\text{when } t=0, w(x, t) = w_0(x) \text{ and } \frac{\partial^4 w}{\partial t^4}(x, t) = w_x(x) \quad (7)$$

and solving the governing equation for the natural frequencies of the beam we get,

$$\omega = \beta^2 \sqrt{\frac{EI}{\rho A}} = (\beta l)^2 \sqrt{\frac{EI}{\rho A l^4}} \quad (8)$$

For the simple support condition,  $(\beta_n l) = n\pi$ , where  $n = 1, 2, 3, \dots$

Equation (8) gives the natural frequencies for the intact condition.

Now consider damaged condition as both spot welds of one of the stiffener ends are sheared off. Being separated plate and hat stiffener are not in firm contact in damaged portion, vibrates independently (neglecting rubbing action). Let the neutral axis position been unchanged. Also, weight of the separated portion of the hat stiffener is supposed to act at the nearest spot welds. Thus the model of the structure is now become like a stepped beam with a step at distance  $l_1=287$  mm from end A or distance  $l_2= 137$  mm from end B as shown in Figure4. Change in M. I. at step is  $I$  to  $0.16I$ .

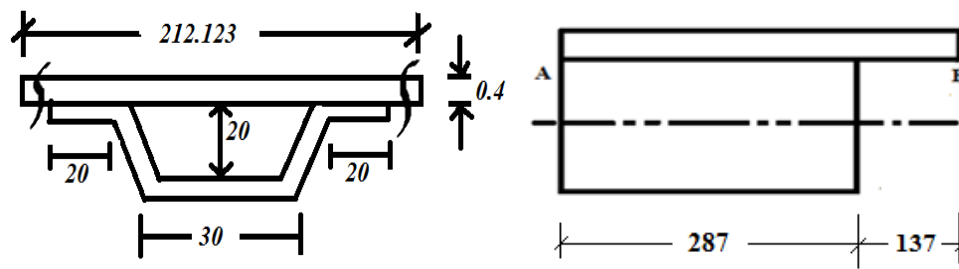


Figure 4: Orthographic Views of Transformed Beam Model for Damaged Condition

The stepped beam is assumed as lumped mass system. The hat stiffened plate is equally divided into four parts along the stiffener as shown in Figure 5.

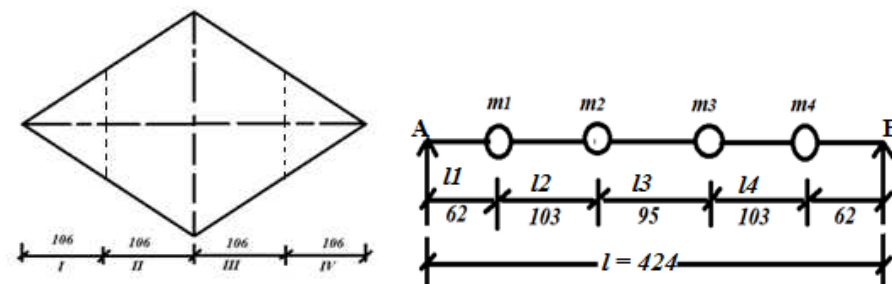


Figure 5: Division of Plate into Four Parts and Lumped Masses with their Locations

Initially the magnitude ( $m_1$ ,  $m_2$ ,  $m_3$ , and  $m_4$ ) and distances from point A ( $l_1$ ,  $l_2$ ,  $l_3$ , and  $l_4$ ) of lumped masses are determined. A conjugate beam method is used to find the static deflections  $u_{ij}$  of mass  $m_i$  at location  $l_i$  ( $i, j = 1, 2, 3, 4$ ), when a unit force  $W_j$  is applied at location  $l_j$ . Thus, all flexibility coefficients  $u_{ij}$  are determined and flexibility matrix is formulated. Let  $a$  and  $b$  be the distances of unit load  $W$ , from points A and B, respectively. Loading diagrams for conjugate beam have been shown in Figures 6-8.

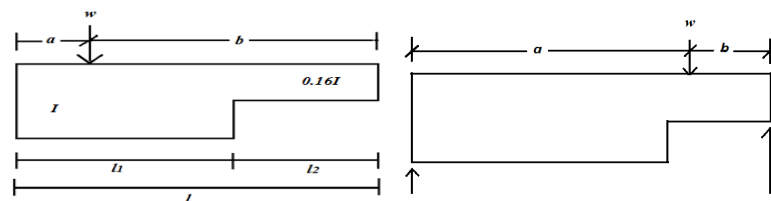


Figure 6a: Actual loading diagram for beam ( $a < l_1$ )

Figure 6b: Actual loading diagram for beam ( $a > l_1$ )

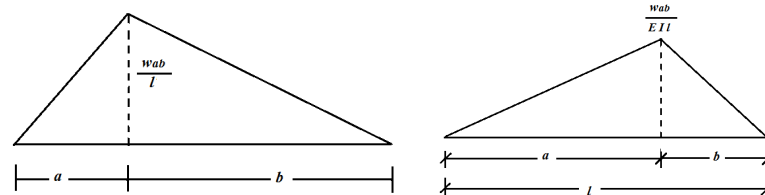
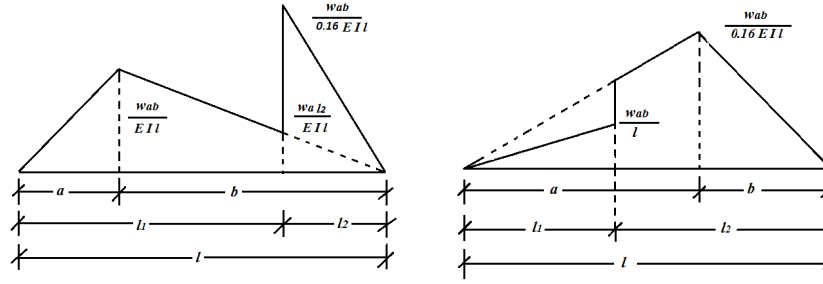


Figure 7a: Bending Moment (BM) Diagram ( $a < l_1$ )

Figure 7b: Bending Moment (BM) Diagram ( $a > l_1$ )



**Figure 8a: Loading Diagram of Conjugate Beam ( $a < l_1$ )      Figure 8b: Loading Diagram of Conjugate Beam ( $a > l_1$ )**

Putting flexibility coefficients matrix A and Mass matrix M in equation of motion,

$$[M][\ddot{x}] + [K][x] = 0 \quad (9)$$

$$\text{Multiplying by } [K]^{-1} \text{ we get, } [K]^{-1}[M][\ddot{x}] + [I][x] = 0 \quad (10)$$

Where  $[K]^{-1}[K] = [I]$  and  $[K]^{-1}[M] = [D]$ , a dynamic matrix

$$\text{Now letting the solution of the form, } x = X \sin \omega t \therefore \ddot{x} = -\omega^2 X \sin \omega t = -\frac{1}{\lambda} X \sin \omega t \quad (11)$$

$$\text{Where } \lambda = \frac{1}{\omega^2}, \text{ then matrix equation becomes, } [D][X] - \lambda [I][X] = 0 \text{ or } [[D] - \lambda [I]][X] = 0 \quad (12)$$

The solution of the above equation is obtained by putting its determinant equal to zero.

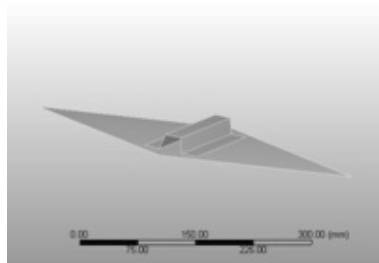
$$|D - \lambda I| = 0 \quad (13)$$

This Equation 13 gives the required natural frequencies for the damaged condition.

### 3. FINITE ELEMENT MODELING

A software package Pro/ENGINEER is used to generate a geometric model (Figure8) of hat stiffened plate. This model is browsed in ANSYS<sup>R</sup>15 workbench and spot weld connections are applied at the specified locations. The boundary conditions are applied at four corners of the plate. Using the modal tool the model is solved by finite element method to obtain natural frequencies for the intact condition

For the damaged condition, both the sheared off spot weld connections are suppressed. This model is again solved to obtain natural frequencies for the damaged condition. Thus the natural frequencies for the intact and damaged hat stiffened pate have been used for the validation purpose.



**Figure 8: Geometric Model of Hat Stiffened Plate**



**Figure 9: Modal Test using FFT Analyzer**

#### 4. EXPERIMENTATION

A square thin plate & hat stiffener both made of steel sheet of thickness 0.4mm is prepared and assembled using sheet metal processes. Here a stiffener is not allowed to merge with the edges of plate, as it reduces the stiffness of plate. Each side flange of the hat stiffener is joined to the plate by 5 equally spaced and approximately 7 mm diameter spot welds applied between them. Thus a notable increment in the strength of the plate is assured due to 10 spot welds applied between plate and stiffener.

Now to apply the simple support at the four corners of the plate, it is held by two separate non-elastic & light weight cord. Both ends of one cord are joined one diagonal ends of square plate whereas another cord is joined to the remaining diagonal ends. Thus when suspended, both the cords cross over each other at pivot as shown in Figure9. To determine the natural frequencies of the hat stiffened plate, it is decided to carry out an experimental modal analysis, for the measurement and analysis of the vibrations an OROS 3-Series/ NVGate. Fast fourier transform (FFT) analyzer set up is used. A tiny piezoelectric accelerometer is selected and placed at the centre on flat side of the plate. A single input and single output (SISO) method of excitation is used for exciting the structure. From the frequency response plots (FRP), as an output of FFT analyzer, the frequencies corresponding to initial consecutive picks is obtained as modal frequencies for further assessment.

To introduce the damage both spot welds on one of the ends of stiffener are sheared off. And the above modal test procedure is repeated. Thus from the new FRP for damaged spot weld condition of the stiffened plate, initial natural frequencies are obtained again.

#### 5. RESULTS AND DISCUSSIONS

Three approaches viz. analytical. FEM and EMA are used to find the natural frequencies of hat stiffened plate assembly for damage detection. Initially the intact structure is considered to have a transformed section. Applying Euler-Bernaulli's equation to intact hat stiffened plate has given initial natural frequencies of flexural vibrations, as shown in Table2. Now, the solution to the matrix equation for linear un-damped system is solved, to obtain the natural frequencies for damaged condition, as given in Table 2. It has been found that, there is a decrement in the natural frequencies due to the occurrence of damage. The difference between the natural frequencies increases with the mode number. After third mode, an absurd result has been observed. This may be due to the coupled flexural and torsional modes, at higher frequencies.

A numerical method using ANSYS software package also rendered the results, for the intact and damaged hat stiffened plate structure, are given in the Table2.

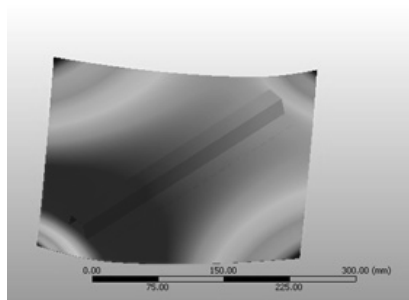


Figure 9: Intact Model (1<sup>st</sup> Mode)

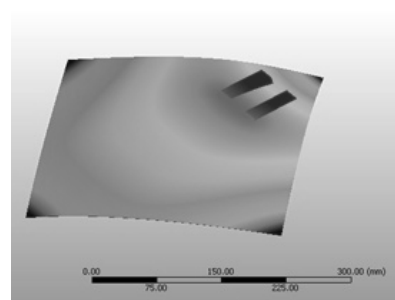
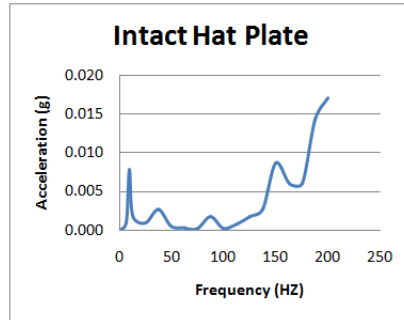
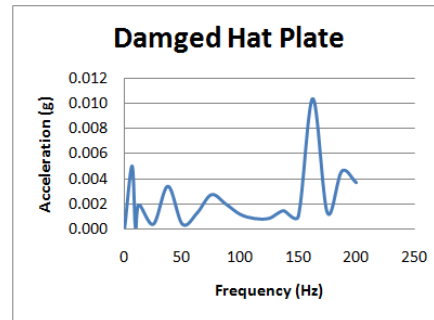


Figure 10: Damaged Model (1<sup>st</sup> Mode)

Here, also damage has caused the reduction in the natural frequency at each mode and thus validates analytical method. The deformed shapes at particular modes are shown in the Figure 9 for the intact model and Figure 10 for the damaged model. In case of both the conditions it can be observed that, at the higher modes, torsional vibrations become more dominants as compared to flexural one.



**Figure 11: Frequency Response Plot for Intact Stiffened Plate**



**Figure 12: Frequency Response Plot for Damaged Stiffened Plate**

Now the experimental modal analysis of hat stiffened plate (0.4 mm thin) for both intact and damaged conditions had rendered the results in the form of frequency response plots. Natural frequencies for each condition were obtained by identifying the consecutive pick occurred in the respective FRPs. Both the FRPs are shown in Figure 11 and Figure 12. The natural frequencies extracted from FRP for intact and damaged conditions are mentioned in Table 2. The decrement in the modal frequencies was related to the separation of stiffener from the plate due to transverse cracking of the weld joints. As the structure was light in weight, it was essential to use accelerometer as tiny as possible.

**Table 2: Comparison of the Results Obtained from Analytical, Finite Element and Experimental Modal Analysis for the 0.4 mm thin hat Stiffened Plate for Intact and Damaged Conditions**

Mode Nos.	Modal Frequencies (Hzs.) Obtained from Analytical Method.		Modal Frequencies (Hzs.) Obtained from Finite Element Method		Modal Frequencies (Hzs.) Obtained from Experimental Modal Analysis	
	Intact	Damaged	Intact	Damaged	Intact	Damaged
1	9.04	7	10.598	7.9957	9.5	6.5
2	36	11	43.214	41.977	37.5	12.5
3	84.38	34.67	97.741	74.108	87.5	37.5
4	144	159	145.73	133.59	150	75

## CONCLUSIONS

Damage of the hat stiffened plate is been identified using the free vibration analysis. The results obtained from analytical, finite element and experimental modal analysis shows that, the natural frequencies of hat stiffened plates with intact and damaged conditions are a function of spot weld cracking. The change in the physical properties, due to spot weld cracking has decreased the global stiffness of the plate and hence, the natural frequencies also. The transformed section method is successfully used, to analyze hat stiffened type complex structure. The mathematical model for the damage has been developed; by considering it as a change in the cross section. The mathematical model for the damage has been developed by considering a change in cross section of the structure. This structure with varying cross section is solved by convenient method, like conjugate beam method.. A software package, for finite element analysis, ANSYS, has reliably solved the model for both intact and damaged conditions. An experimental modal analysis (EMA) can be used, as one of the NDT methods, for damage detection in structures, provided it needs careful interpretation. While experimentation,



coincidentally, same frequency occurs for intact and damaged models also. The remedy to this confusion is to note the mode number. Only initial modes are required to detect change in the physical properties like stiffness.

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